

SOME ASPECTS OF THE LOW-SPEED AND SUPERSONIC AERODYNAMICS OF LIFTING SLENDER WINGS

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ABSTRACT

The design of a low-aspect ratio, delta-like lifting surface for a supersonic transport aircraft is described, within principles aimed at maintaining a single well-behaved flow under all flight conditions. The wing is to have low lift-dependent drag and be trimmed at cruise about a centre of gravity providing longitudinal stability at low speeds. Supersonic wind-tunnel results on three models, including one with a short forebody, show the success of the method.

Results of low-speed experiments are reviewed to show the lift available for approach, the pitching moment and drag characteristics and the nature of the flow development on such configurations.

INTRODUCTION

At the Second I.C.A.S. Congress in Zürich, Küchemann¹ gave a brief survey of the aerodynamics of three shapes suitable for supersonic flight. Among these, he included the slender wing of near-triangular planform cruising with subsonic leading edges and supersonic trailing edge, the leading edges being in general curved and having streamwise tips. In order to insure a steady and stable flow associated with the regular development of leading edge vortex sheets, he demanded aerodynamically sharp edges and a distribution of camber and thickness such that at some chosen incidence the flow would separate only from the trailing edge, as described by Maskell and Weber.² He could only touch on the problem of matching cruising and low speed requirements, and did not attempt to discuss trimming at cruise.

In the present paper we continue the discussion of the aerodynamic design of slender wings, accepting the basic principles stated by Küchemann, Maskell, and Weber. In addition we shall bear in mind the need to provide natural static stability over the whole range of speed and incidence and therefore assume that the centre of gravity of the aircraft will be at, or just ahead of, the low-speed aerodynamic centre. At this point we encounter a significant difference between the flow past a slender wing at low speeds and at supersonic speeds. The load no longer vanishes at the trailing edge at supersonic speeds and the aerodynamic center moves aft, creating a problem of trim. To deal with this we introduce lengthwise camber so that the centre of lift at cruise is at the centre of gravity, so that the aircraft is trimmed at cruise without control deflection and consequent drag penalty. Attached flow is maintained at the leading edges by an appropriate distribution of crosswise camber. We employ linear theory to obtain the required camber surface, and combine this with a thickness distribution intended to provide sufficient volume in the right places to make the wing usable for a transport aircraft. The results of supersonic wind-tunnel tests on two ogee wings, and on a third model having a round body over the first third of its length, are quoted to show the success achieved in the cruise condition.

In all of this we have kept the requirements of low-speed aerodynamics in mind, in our choice of span-length ratio and of a smooth leading-edge planform, and in avoiding an excessively drooped leading edge. From tests on an earlier series of models and on a set similar to the ones referred to a moment ago, we shall discuss what we know at present about the limitations imposed by low-speed aerodynamics on wing shape. We shall review the flow development and the lift, pitching moment and drag characteristics with special attention to the effects of camber and of a short round body protruding ahead of the wing. Flight dynamics and lateral stability characteristics are beyond our present scope.

At this point, and before embarking on this program, the authors acknowledge the extent of their indebtedness to many members of the Aerodynamics Department at Farnborough and Bedford. It will be apparent that a considerable program of work has been done, the results of which have been freely incorporated in this paper without individual acknowledgment.

CAMBER DESIGN FOR CRUISE

Let us now suppose that we have arrived at a planform and volume distribution and we wish to design a camber surface for it. Since we have chosen to preserve natural longitudinal stability at low speeds, and since the aerodynamic center of the wing moves back between subsonic and supersonic speeds, we face the problem of restoring the center of lift at cruise to the centre of mass. Figure 1 shows the displacement of the aerodynamic center from its most forward position at the approach to its position at cruise. We see that this displacement is more or less constant, so the problem is independent of the planform.

We do not propose to discuss the various methods that might be used to deal with this. It seems obvious that, if it can be done by a smooth camber distribution without drag penalty, then that is the way to do it.

The first requirement of our camber surface is, then, a given centre of pressure some 8 percent of the wing length ahead of the cruise aerodynamic centre. This implies lengthwise camber with the wing apex at a larger incidence than the trailing edge. Such camber alone would violate our design principles; vorticity might be shed from different parts of the leading edge towards the upper and lower surfaces simultaneously, introducing the possibilities of instability and unsteadiness associated with free vortex sheets. The leading edge must therefore be bent down into the local flow direction to make the leading edge an attachment line at some incidence, which need not be the incidence at which the aircraft cruises. The load must vanish there at this "attachment" incidence and the appropriate behavior is that it vanishes like the square root of the distance. This is the second requirement on our camber surface. We must now consider the third requirement of low drag, although we have no intention of "optimizing" the design. At the attachment incidence we have a flow with genuinely small disturbances. By choice of the pressure distribution we keep the boundary layer attached and avoid upstream influence of the trailing-edge shock system. We may then confidently apply the linearized theory of supersonic flow.

According to this, we can separate the drag into that of the uncambered wing and that of the camber surface. The latter can again be divided into vortex drag and lift-dependent wave drag. The vortex drag can be reduced by making the spanwise distribution of chord-loading as nearly elliptic as possible within the limitations which off-design conditions impose on the wing. The

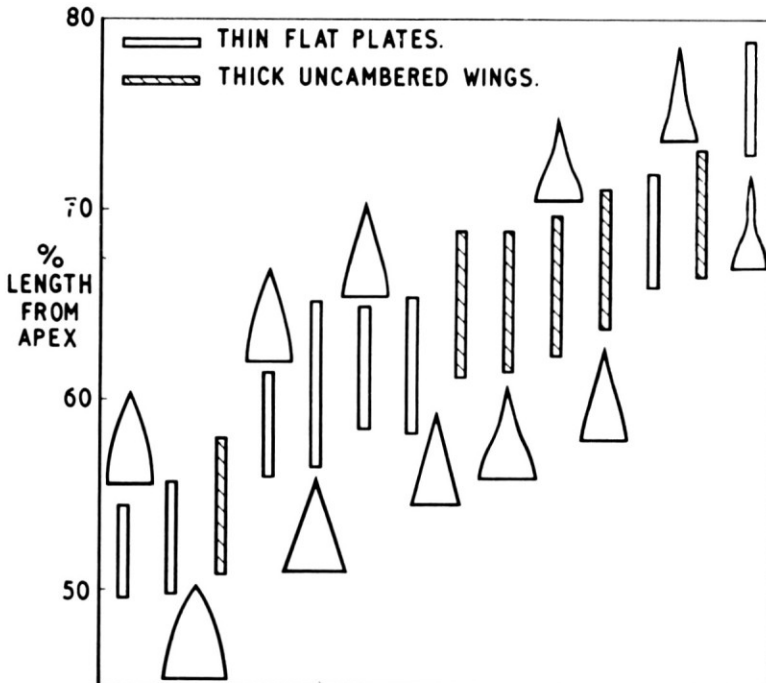


Fig. 1. Shift of aerodynamic centre between $M \approx 0.3$, $C_L = 0.5$, and $M \approx 2$, $C_L = 0.1$.

lift-dependent wave drag depends on the entire distribution of load over the wing, but, for a sufficiently slender wing, it is mainly governed by the lengthwise distribution of cross-loading. For a low drag, this should be smooth, with a low peak not too far forward or aft, maintaining a fairly large value at the trailing edge. If our attachment incidence is also our cruise incidence we can now express the third requirement of low lift-dependent drag as a condition on the load distribution of the wing.

Since for a specified distribution of volume, the pressure field in which the boundary layer develops is also given in terms of the load distribution, it is natural to proceed by designing a load distribution and calculating the corresponding camber surface. As already stated, we feel justified in applying supersonic linearized theory, and find the standard expressions which (Fig. 2) involve improper integrals. This means that their numerical evaluation involves difference operations as well as summations. Happily for the numerical analyst, they can be recast so as to involve summation processes only, in the case when the load is given analytically and vanishes along the leading edge.

We find experimentally that a large leading-edge droop reduces the non-linear lift at the approach and that the lift-dependent drag factor, obtained by dividing the drag by the square of the lift, is usually lower at cruise above the attachment incidence. We therefore wish to adapt our design procedure to the case in which the attachment incidence is below the cruise incidence.

Once the cruising aerodynamic center is known, we can choose the load distribution at the attachment incidence so that the required center of pressure is obtained at the higher cruising incidence. The drag at cruise cannot readily be calculated, since the flow is separated, so we rely on two plausible principles. The spanwise distribution of chord-loading due to a small leading-edge separation is similar to that for which we design, so the vortex drag should be

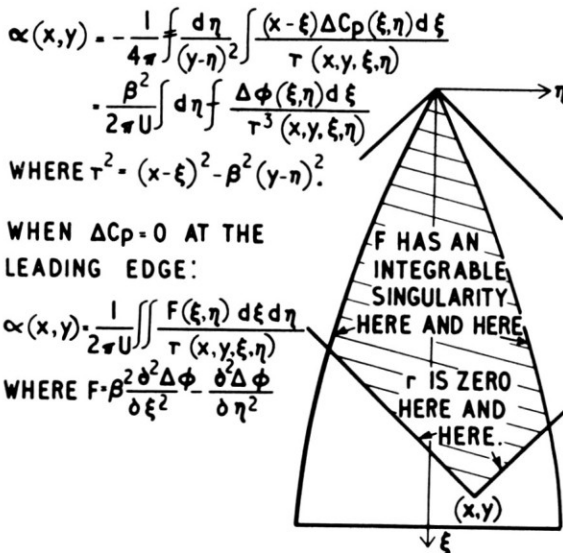


Fig. 2. Formulae derived from supersonic linearized theory.

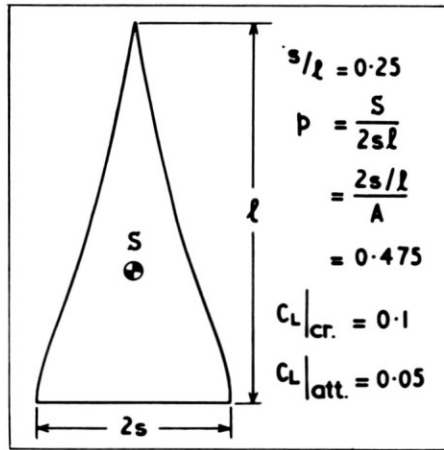


Fig. 3. Planform of cambered wings.

little altered by a small increase of incidence above attachment. The lengthwise distribution of cross-load due to additional incidence must differ essentially from that due to the camber surface, but we suppose that the combination of the two still governs the remainder of the lift-dependent drag. We must then shape the total lengthwise load distribution as already described.

Let us now see how all this works out in practice. Figure 3 shows a slender wing with its centre of mass marked at the estimated position of the low speed aerodynamic centre at the approach. It has a semispan to length ratio of 0.25 and its area occupies 0.475 of the rectangle which just encloses it. This latter ratio will be denoted by p . Its relation to aspect ratio and semispan to length ratio is obvious. We propose to design a warp distribution for a lift coefficient of 0.05, intended for an aircraft to cruise at a lift coefficient near 0.1 at a Mach number of 2.2. We first choose a lengthwise distribution of cross-load which has the cruising lift coefficient and the cruising centre of pressure position and which satisfies the conditions we suggested for low lift-dependent wave drag. This is shown in Fig. 4. We must then subtract from this the lengthwise distribution of cross-load due to a lift coefficient of 0.05 on the corresponding flat wing, in order to get the load distribution at attachment. This was estimated for the present purpose. We now choose for the variation of the load in the spanwise direction a behavior which concentrates the downward inclination of the wing into a region near the leading edge. A kind of "shoulder-line" results which is chosen to be relatively further inboard near the wing apex than near the trailing edge so as to combine nearly elliptic chord-loading with a smooth shape near the apex. The wing which results when a suitable thickness distribution is added to the calculated mean surface is shown in Fig. 5.

When this wing was tested at the design Mach number it produced the design lift coefficient within one-tenth of a degree of the attachment incidence. At the design lift coefficient it produced between 90 and 95 percent of the design pitching moment, corresponding to a centre of pressure less than 1 percent of the length from the design position. The shift in aerodynamic centre between a

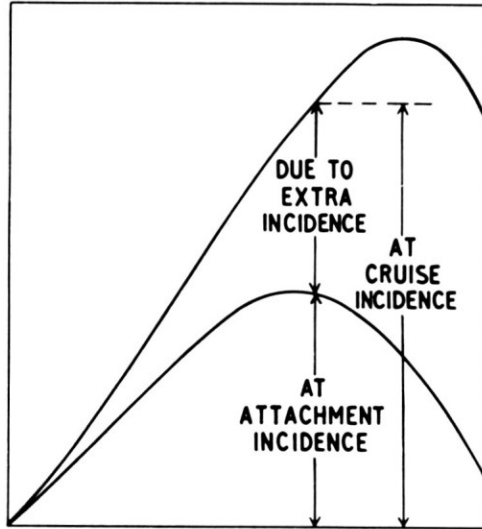


Fig. 4. Distribution of cross-loading of first cambered wing.

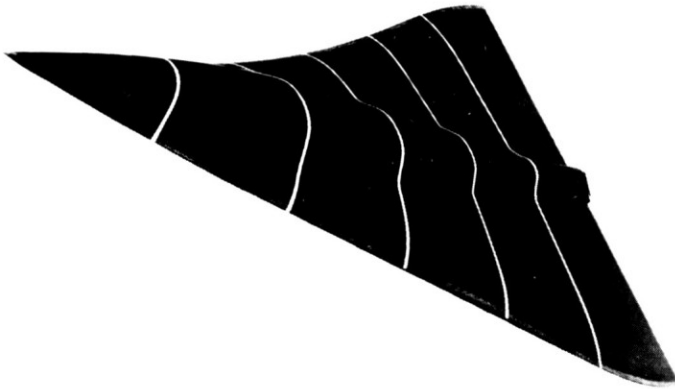


Fig. 5. First cambered wing.

Mach number of 2.2, lift coefficient 0.05, and low speed, C_L of 0.5, was almost exactly 8 percent of the length, as assumed in the design.

However, the aerodynamic centre at low speeds of the plane wing was about 1 percent further forward than we had estimated and the aerodynamic centre of the cambered wing was about 1 percent further forward still. Consequently, the wing trimmed at a lift coefficient 0.08 instead of 0.1. The lift-dependent drag factor as defined earlier was marginally lower than that of the plane wing at the cruise Mach number and lift coefficient. (Fig. 6.)

Results very similar to these were obtained with a wing of the same planform designed to have attached flow at C_L zero. This wing produced slightly less than 90 percent of the Cm_0 it was supposed to and the trimmed C_L was a little larger than before. The lift-dependent drag factor was again close to that of the plane

wing at the cruise condition, but this time it was increasing with C_L and lay above that of the flat wing at lower Mach numbers.

Essentially the same results have been obtained on a configuration with a near-circular body protruding forward of the wing for about a third of the length. The semispan to length ratio was again 0.25 and the gross plan area was 0.45 of the enclosing rectangle. The wing was designed with the additional restriction that the local incidence of the wing apex was to vanish at the attachment incidence. The body was then added so that its centre line followed the warped centre line of the wing and was carried straight on into the wind direction at the attachment incidence. The combination in fact again trimmed at 80 percent of the intended cruise C_L and had marginally less drag at the cruising condition than the corresponding unwarped configuration.

These experimental results (summarized in Fig. 7), taken together, demonstrate a remarkable degree of success from the use of small disturbance theory.

LOW-SPEED AERODYNAMICS

We now turn to low-speed aerodynamic considerations. As we stated in the Introduction, the matching of the needs of cruise and low-speed flight has been borne in mind throughout the work. Our purpose is now two-fold; firstly to survey the low-speed aerodynamics of slender wings with particular reference to flow development and longitudinal forces and moments; secondly, to present results of experimental work on wings cambered as described in order to find whether low-speed requirements impose any limitations on the camber design.

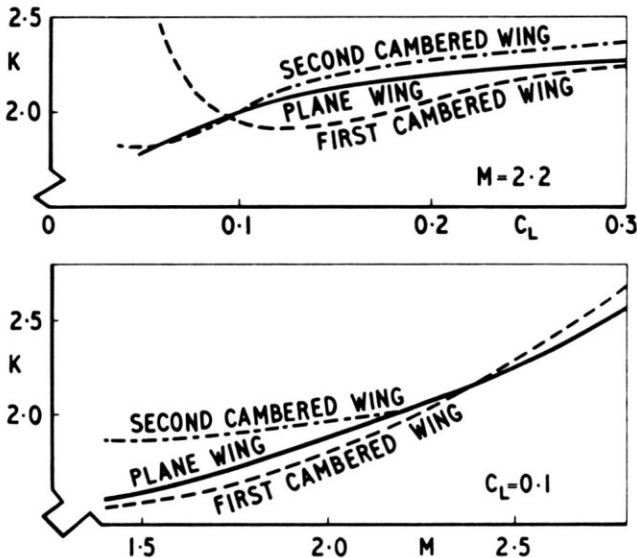


Fig. 6. Lift-dependent drag factor $K = \pi A C_{D_i} / C_L^2$ where $C_{D_i} = C_D - C_{D_0}$ (plane wing).

	FIRST CAMBERED WING	SECOND CAMBERED WING	WING-BODY COMBINATION
DESIGN PARAMETERS:			
ATTACHMENT C_L	0.05	0	0.05
CRUISE C_L	0.1	0.1	0.1
C_m AT ATTACHMENT C_L (ABOUT NOSE IN TERMS OF LENGTH.)	-0.0283	0.0080	-0.0303
MEASURED VALUES AT $M = 2.2$:			
C_m AT ATTACHMENT C_L	-0.0288	0.0071	-0.0310
TRIMMED C_L	0.08	0.08	0.08
K AT $C_L = 0.1$	1.95	2.02	1.85
K OF UNCAMBERED CONFIGURATION AT $C_L = 0.1$	2.02	2.02	1.94

Fig. 7. Summary of supersonic tunnel results.

FLOW CHARACTERISTICS

It has been evident for a long time that thin wings of the kind of planform we are considering give regular development of the leading edge vortex sheets and a stable, steady flow, provided that the wing is uncambered or designed for attached flow at some low incidence, and provided also that the planform is smooth with only gradual changes of sweepback along the leading edge. The need to avoid "vortex breakdown," with its unsteady flow and buffeting, does not impose too severe a limitation, since with leading edge sweepback of 70° , vortex breakdown will remain downstream of the wing unless the incidence exceeds about 30° . For such thin wings then, good flow is obtained on a large range of shapes, including the parabolic gothic of $p = \frac{2}{3}$, and ogees of $p = 0.45$ or less with gradual variations of leading edge sweepback.

Thickness and camber suitable for transport aircraft must now be discussed. Recently, two series of cambered wings have been tested (Fig. 8) with $p = 0.45$, $s/l = 0.208$ and $p = 0.475$, $s/l = 0.25$, the second being the same as those in the supersonic tests discussed earlier. These include highly cambered wings with lift coefficients for attached flow up to 0.075 and loadings giving C_m at zero lift up to 0.014. Also included is the wing-body configuration discussed in the earlier part of the paper.

These models had attached flow everywhere at low speed at incidences very close to the $M = 2.2$ design points, and the only worries about the flow apply near the nose at moderate incidences. On the wings without bodies, very thick cross sections with large edge angles and large droop near the edge may give attached flow or very weak separation at the edge, sometimes with separations further inboard leading to what may be called body vortices. Further work is

needed to define the range of shape near the nose within which this behavior is avoided.

On the wing-body configurations, the upwash alongside the body insures that the origin of the wing leading edge separation is firmly anchored, and good development of the wing flow results. Body vortices are shed above the nose, and, though these give no asymmetry to the flow below about 30° incidence when the nose is so short, they do get pulled into the wing vortices and rolled round them. Further work is here needed, particularly on dynamic motions of wing-bodies, to define what is acceptable.

LIFT

Our second low-speed requirement is the provision of sufficient lift for the approach condition. Figure 9 shows the lift coefficient at 15° incidence on uncambered wings and wing-body configurations. Untrimmed lift is plotted against semispan/centre line chord as this seems to be a more successful parameter than the aspect ratio. It is beyond our scope to discuss project studies, except to say that for a transatlantic transport with about 130 seats these studies based on wings such as the example used in the first part of this paper suggest that a lift coefficient of about 0.5 will be sufficient to permit an approach speed of 140 knots and enable existing runways to be used. The results shown in the figure account then for our choice of a semispan/length of 0.25 in our main example, and here we would stress the virtues of this choice which results in a desirable

MODEL	p	$\frac{s}{\ell}$	$C_{L \text{ ATT.}}$	DESIGN $10^4 \Delta C_m$	EDGE DROOP		REMARKS
					$\theta^\circ_{0.4\ell}$	$\theta^\circ_{0.9\ell}$	
1	0.45	0.208	—	—	—	—	SYMMETRICAL
2	"	"	0	85	28	1	
3	"	"	0	116	49	1	
4	"	"	0	116	49	1	
5	"	"	0	136	58	1	
6	"	"	0.025	85	50	12	
7	"	"	0.025	67	30	14	
8	"	"	0.050	49	32	25	
9	"	"	0.075	30	34	34	
10	0.475	0.25	—	—	—	—	SYMMETRICAL
11	"	"	0	128	26	1	
12	"	"	0.05	128	42	14	
13	"	"	0.05	128	42	24	
14	0.45	0.25	—	—	—	—	SYMMETRICAL 14 & 15 ARE WING-BODY CONFIGURATIONS
15	"	"	0.05	107	27	13	

NOTE: 2-9 DESIGNED BY SLENDER THEORY.
11-13 AND 15 BY LINEAR THEORY AT $M=2.2$.

Fig. 8. Details of cambered models tested at low speed.

flow over an incidence range giving an adequate margin above that for the approach, and at the same time gives sufficient lift for our purpose. Figure 9 also shows that ogee wings give rather more lift than gothic or delta wings (or alternatively that the effective length of an ogee is some 15 percent less than its true length in respect of lift), and that adding a forebody generally causes a small reduction of lift.

The effects of camber on trimmed lift are shown in Fig. 10. The upper results show that camber of the type discussed causes a reduction in the lift at an incidence 15° above that for zero trimmed lift. The loss is 0.02 to 0.03 in all cases and does not seem to vary with any particular feature of the camber. The lower set of results shows that if a line joining 70 percent centerline chord to the trailing edge is set at 15° , representing the use of a fixed length of undercarriage, then increasing the lift coefficient for attached flow, $C_{L_{att}}$, decreases the available low-speed lift. As we shall show later, however, the drag due to lift at low-speed is decreased, so that a compromise choice of the value of $C_{L_{att}}$ must be made.

PITCHING MOMENTS, AERODYNAMIC CENTRE AT LOW SPEED

The next important aerodynamic feature of slender wings at low speed is the aerodynamic centre. Essentially because we are discussing a tailless layout, the difference between the maneuver margin and the static margin is small, and so

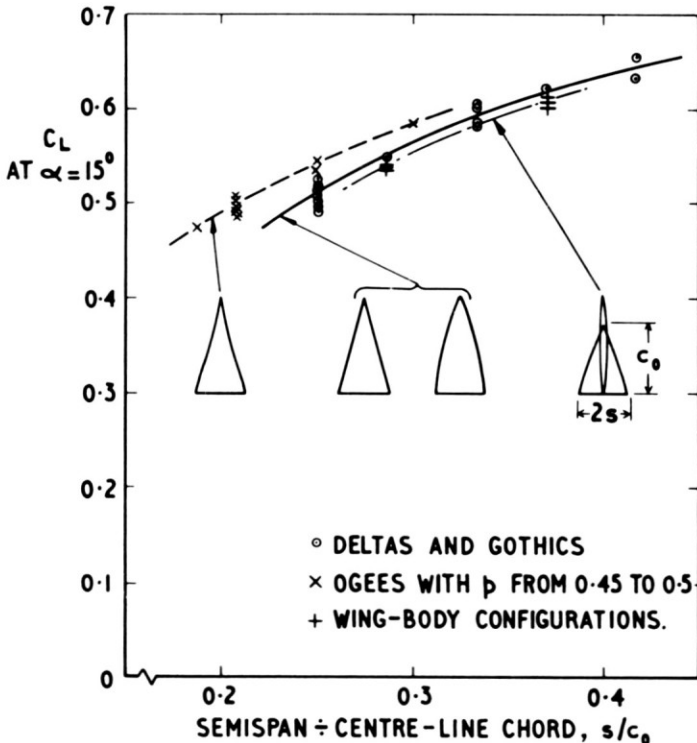


Fig. 9. Lift at 15° incidence, uncambered wings with t/c_0 0 to 0.08.

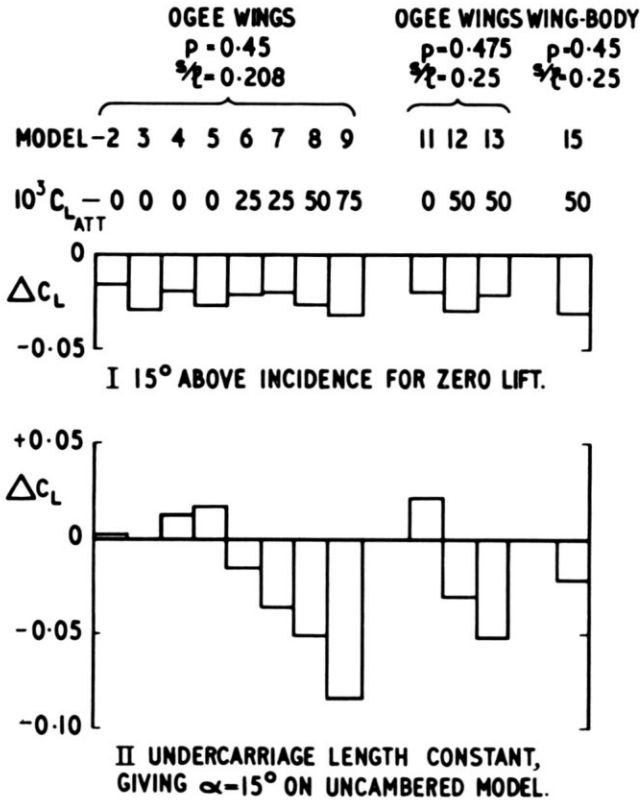


Fig. 10. Change in trimmed lift at 15° incidence due to camber ($M \approx 0.15$).

the furthest forward position of the low-speed aerodynamic centre fixes the furthest aft position of the centre of gravity. For most wing planforms we are discussing, except those of high ρ values like the gothic, the low-speed aerodynamic center moves slowly forward as incidence increases, so that the condition of highest incidence, the approach, defines the centre of gravity. It is argued that slight static instability may be tolerated at higher incidences, though one must insure that no rapid forward movement of aerodynamic centre occurs.

Figure 11 shows the aerodynamic centre at a lift coefficient of 0.5 for uncambered wings and wing-body configurations plotted simply against the fore-and-aft position of the centre of area of the wing. In this simple diagram, first used by Handley Page Ltd., the experimental results are remarkably close to a straight line, though it is seen that increasing thickness moves the aerodynamic centre back, and more detailed examination of the original data shows that increasing aspect ratio has a small effect in the opposite direction. The curve also shows that the addition of a body nose ahead of the wing has little effect.

Figure 12 shows the effect of camber on the aerodynamic centre. Theory suggests that the local lift slope increases with the droop angle at the leading edge in cross section. The difference between the tangent of the droop angle at 0.4 l and 0.9 l (equal distances ahead of and behind the aerodynamic center) has

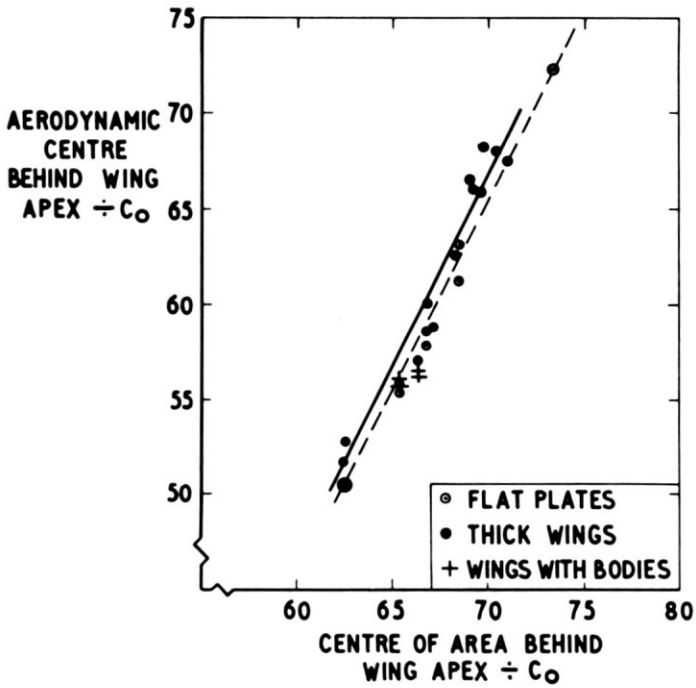


Fig. 11. Aerodynamic centre at $C_L = 0.5$ and low speed.

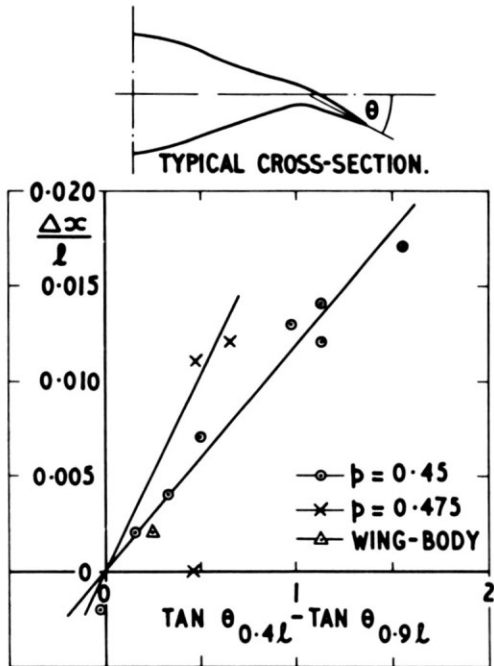


Fig. 12. Forward movement of aerodynamic centre caused by camber, at $C_L = 0.5$ and low speed.

therefore been taken as a simple indicator of the expected effect of camber, and is seen to collect the results for the series of wings of $\rho = 0.45$ on to a straight line. The crudity of using droop angles at two arbitrary points is shown by the results for the models of $\rho = 0.475$ which had load distributions giving much larger droops near the nose. It is not surprising that they give larger movements of the aerodynamic center. The gull-wing model has the same aerodynamic centre as the symmetrical wing and the reason for this is not understood.

DRAG DUE TO LIFT AT LOW SPEED

In the application of slender wings to supersonic transport aircraft, low-speed drag is a fairly important matter, since roughly a tenth of the all up weight of the aircraft consists of fuel for holding and diversion phases. To complete our survey of the longitudinal characteristics of these wings at low speeds, we have collected in Fig. 13 the induced drag factor $\pi A(C_D - C_{D_0})/C_L^2$ where C_{D_0} is the minimum drag of the corresponding uncambered wing. The values are taken at a lift coefficient of 0.2; a similar picture though with more scatter would be obtained at lower lift more nearly corresponding to maximum lift/drag ratio.

Two points are worth comment. The first is that if we fix the semispan/length ratio and vary the planform parameter ρ , the resulting change in aspect ratio scarcely changes the induced drag because it is almost completely offset by the change in the drag factor. And secondly, although camber giving attached flow at zero lift has little effect, increase of the lift coefficient for attached flow causes a marked reduction in the induced drag at low speed, at any rate up to $C_{Latt} = 0.075$.

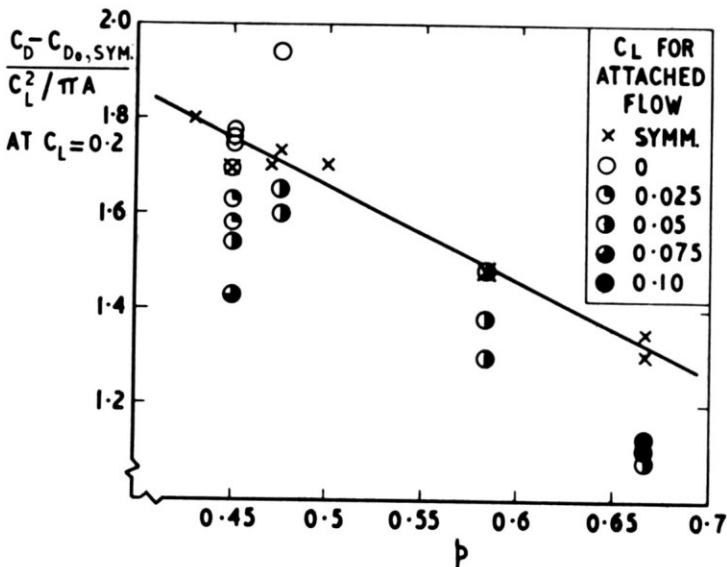


Fig. 13. Low-speed lift-dependent drag.

CONCLUDING REMARKS

In conclusion, we have described a continuation of aerodynamic research on slender wings following the paper presented by Küchemann at the last I.C.A.S. Congress. We have shown that a near-delta wing, with or without a round body protruding at the nose, can be cambered so as to be trimmed for supersonic cruise about a centre of gravity giving positive static longitudinal stability over the whole speed range. Examples have been given, showing that such wings have low lift-dependent drag. With a planform chosen to suit low-speed and high-speed requirements, we have shown that low-speed aerodynamics impose no severe restrictions on the camber and thickness distribution except near the nose. Here, more work is needed to show what is acceptable, whether the wing is continued with sharp edges over the whole length, or whether a round body protrudes. With this minor proviso, we consider that slender configurations as discussed today can serve as efficient, trimmed, supersonic aircraft with adequate lift for the approach and natural longitudinal stability throughout the speed range. The same steady and stable type of flow persists throughout so that the slender wing offers the designer the first really natural alternative to the classical aircraft.

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DISCUSSION

Authors: A. Spence and J. H. B. Smith

Discussor: G. H. Lee, Handley Page Ltd.

Some low-speed wind-tunnel tests carried out by Handley Page a few years ago on a special type of slender delta may be of interest as showing the way in which camber may reduce the induced drag.

We made a model as shown below:

The first 70 percent of the wing had a cross section like *AA*; this was faired-off at the back.

The idea was to produce flat surfaces, such as *XX*, the normal to which had a component in the forward direction. The sharp leading edges produced strong L.E. vortices which lay above these flat surfaces. The suction from the vortices on these surfaces was therefore partly directed forward and so reduced induced drag.

Tests showed that in some cases the induced drag factor was little more than half that of the corresponding uncambered model; usually the reduction was to 70 or 80 percent of the uncambered wing value. In a typical case at $C_L = 0.3$, the total drag was about 80 percent of that of the uncambered standard model.

The model tested was not of a very practical shape, but the tests and results throw some light on the way in which camber may reduce induced drag.

Author's reply to discussion:

I should like to thank Mr. Lee for his contribution. I agree with him entirely on the suggested mechanism by which camber reduces the drag due to lift, and would only wish to add that thickness also plays an important part in the same way, causing the induced drag of a thick, uncambered wing to be very much less than that of a thin flat plate.

Discussor: Freidrich Kowalke, EWR-Süd, Munich, Germany

The development of the English supersonic transport was first foreseen in terms of a slender wing in its pure form. Only later did one return to a conventional wing-body combination—probably for practical reasons. It is well known that the lift/drag ratio of a wing is worsened by the addition of a fuselage. If one wishes to have no reduction of L/D , a body-camber must be chosen that gives a desirable type of lift distribution along the length. My question is, "How far does experiment give the theoretically predicted improvement from the optimally cambered body?"

Authors' reply to discussion:

I should like first to make it clear that the results reported here of tests at supersonic speeds all relate to configurations which already have the necessary stowage space for use as long-range passenger-carrying aircraft. No additional fuselages are needed. It is true that the addition of a body to a properly designed slender wing results in a fall in lift-drag ratio and so does the replacement of the fore-part of the wing by a round body. In the wing-body configurations tested, where the body grew out of the wing thickness distribution, the intention was that the body should interfere as little as possible with the development of the wing lift at the attachment incidence, though some lift would be carried on the body at cruise. This lift was intended to form a smooth continuation of the lift distribution along the length of the wing.

We have not tried to optimize the distribution of lift along the body because there is little to be gained from this in practice. The round body is a difficult shape on which to produce a chosen distribution of lift of significant magnitude and, if the body protrudes far from the wing root, it would be necessary to droop the nose in order to reduce flow problems at low speeds, giving an unfavorable down-load at cruise.